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CROSSED PRODUCTS OF LOCALLY C^*-ALGEBRAS

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ABSTRACT. The crossed products of locally C^*-algebras are defined and a Takai duality theorem for inverse limit actions of a locally compact group on a locally C^*-algebra is proved.

1. Introduction. Locally C^*-algebras are generalizations of C^*-algebras. Instead of being given by a single C^*-norm, the topology on a locally C^*-algebra is defined by a directed family of C^*-seminorms. In [**9**], Phillips defines the notion of action of a locally compact group G on a locally C^*-algebra A whose topology is determined by a countable family of C^*-seminorms, and also defines the crossed product of A by an inverse limit action

$$\alpha = \lim_{\substack{\leftarrow \\ n}} \alpha^{(n)}$$

as being the inverse limit of crossed products of A_n by $\alpha^{(n)}$. In this paper, by analogy with the case of C^*-algebras, we define the concept of crossed product, respectively reduced crossed product of locally C^*-algebras.

The Takai duality theorem says that if α is a continuous action of an Abelian locally compact group G on a C^*-algebra A, then we can recover the system (G, A, α) up to stable isomorphism from the double dual system in which $G = \widehat{\widehat{G}}$ acts on the crossed product $(A \times_\alpha G) \times_{\widehat{\alpha}} \widehat{G}$ by the dual action of the dual group. In [**3**], Imai and Takai prove a duality theorem for C^*-crossed products by a locally compact group that generalizes the Takai duality theorem [**12**]. For a given C^*-dynamical system (G, A, α), they construct a "dual" C^*-crossed product of the reduced crossed product $A \times_{\alpha,r} G$ by an isomorphism

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