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ON AN ADDITIVE REPRESENTATION ASSOCIATED WITH THE L_1-NORM OF AN EXPONENTIAL SUM

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ABSTRACT. Let N be a large positive integer parameter, $f(n)$ be an integer valued strictly increasing function of the natural argument n. It is well known that a nontrivial upper bound estimate for the number of solutions of the diophantine equation

$$f(x)+f(y)=f(u)+f(v), \quad 1\le x,y,u,v\le N$$

has an important application in obtaining a lower bound for the L_1-norm of an exponential sum. In this paper by a short argument we obtain a result which implies a well-known estimate of Konyagin.

1. Introduction. Let N be a large positive integer parameter, $f(n)$ a strictly increasing integer-valued function of the integer argument $n, 1\le n\le N$. A famous Littlewood conjecture states that

$$\int_0^1 \left|\sum_{n=1}^{N}\exp(2\pi i\alpha f(n))\right| d\alpha \gg \log N. \tag{1}$$

This conjecture was independently proved in 1981 by Konyagin [**5**] and McGehee, et al. [**7**]. The relation [**8**, page 67]

$$\int_0^1 \left|\sum_{n=1}^{N}\exp(2\pi i\alpha n)\right| d\alpha = \frac{4}{\pi^2}\log N + O(1)$$

shows that the order $\log N$ in (1) is sharp. However, for a wide class of sequences $f(n)$, the estimate (1) can be improved. Bochkarev [**1**] has improved (1) for sequences of the type $f(n)=[e^{Ax^\beta}]$, where $0<\beta<1$.

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