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OSCILLATION THEOREMS RELATED TO AVERAGING TECHNIQUE FOR DAMPED PDE WITH p–LAPLACIAN

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ABSTRACT. We present some oscillation theorems related to integral averaging technique for damped PDEs with p-Laplacian

$$(E)\qquad \sum_{i,\,j=1}^{N} D_i[a_{ij}(x)\|Dy\|^{p-2}D_jy] + \langle b(x), \|Dy\|^{p-2}Dy\rangle + c(x)f(y) = 0.$$

The results obtained extend the criteria for the Sturm-Liouville linear equation due to Kamenev, Kong, Philos and Wong to equation (E).

1. Introduction. We consider the second order damped partial differential equation (PDE) with p-Laplacian

$$(1.1)\qquad \sum_{i,\,j=1}^{N} D_i[a_{ij}(x)\|Dy\|^{p-2}D_jy]+\langle b(x), \|Dy\|^{p-2}\,Dy\rangle+c(x)f(y)=0$$

in an exterior domain $\Omega(r_0) := \{x \in \mathbf{R}^N : \|x\| \geq r_0\}$, where $r_0 > 0$, $x = (x_i)_{i=1}^N \in \mathbf{R}^N$, $N \geq 2$, $p > 1$, $D_iy = \partial y/\partial x_i$, $Dy = (D_iy)_{i=1}^N$, $\|\cdot\|$ and $\langle\,\cdot,\cdot\,\rangle$ denote the usual Euclidean norm and the usual scalar product in $\mathbf{R}^N$, respectively.

Throughout this paper, we assume that the following conditions hold.

(A1) $A = (a_{ij}(x))_{N\times N}$ is a real symmetric positive define matrix function with $a_{ij} \in C_{\text{loc}}^{1+\mu}(\Omega(r_0), \mathbf{R})$, $0 < \mu < 1$.

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