

# DISTRIBUTION OF THE ZEROS OF THE SOLUTIONS OF HYPERBOLIC DIFFERENTIAL EQUATIONS WITH MAXIMA

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ABSTRACT. In this paper, the hyperbolic differential equation with maxima of the form

$$u_{tt} - [\Delta u + \mu(t)\Delta u(x, t - \tau)] + c\left(x, t, u(x, t), \max_{s \in [t-\sigma, t]} u(x, s)\right) = f(x, t),$$

where  $\Omega$  is a bounded domain in  $R^n$  and  $\tau, \sigma = \text{const} > 0$ , are considered. Sufficient conditions for existence of zeros of the solutions of the problems considered in bounded domains are obtained.

**1. Introduction.** In the last few decades, great attention has been paid to automatic control systems and their applications to computational mathematics and modeling [15]. Today, scientists around the globe are showing an increasing interest in differential equations which contains the maxima operator. In many applications the maxima can arise when the control theory corresponds to the maximal deviation of the regulated quantity. For example, neutral hyperbolic and parabolic equations with maxima were investigated in [18–20]. Ladas, Gyor, Bainov and Mishev developed and worked on oscillation theory for the differential equation with delay as in [4, 14] that led to maxima operator applications. Differential equations with maxima have appeared in various systems such as the system

$$(i) \quad u'(t) = -\delta u(t) + p \max_{t-h \leq s \leq t} u(s) + f(t),$$

where  $\delta$  and  $p$  are positive constants. This system has appeared in the theory of automatic control in [15] and the references therein. The

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