ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 37, Number 3, 2007

NONOSCILLATORY SOLUTIONS OF HIGHER ORDER NONLINEAR NEUTRAL FUNCTIONAL DIFFERENTIAL EQUATIONS

YONG ZHOU

ABSTRACT. Consider the forced higher order nonlinear neutral functional differential equation

$$L_n(x(t)+cx(t-\tau))+F(t,x(\sigma(t)))=g(t), \quad t\geq t_0.$$

We obtain a global result, with respect to c, which are some sufficient conditions for the existence of a nonoscillatory solution of the above equation. Our results improve essentially and extend a number of existing results.

1. Introduction. Consider the forced higher order nonlinear neutral functional differential equation

$$L_n(x(t)+cx(t-\tau))+F(t,x(\sigma(t)))=g(t), \quad t\geq t_0, \tag{1}$$

where

$$\begin{aligned} &L_0x(t)=x(t),\\ &L_kx(t)=\frac{1}{r_k(t)}\,(L_{k-1}x(t))', \quad k=1,2,\dots,n \left(' = \frac{d}{dt}\right), \end{aligned}$$

$r_k:[t_0,\infty)\to(0,\infty)$, $k=1,2,\dots,n-1$, $r_n\equiv 1$, $\sigma, g:[t_0,\infty)\to\mathbf{R}$, and $F:[t_0,\infty)\times\mathbf{R}\to\mathbf{R}$, $t_0\geq 0$, are continuous, $\sigma(t)\to\infty$ as $t\to\infty$.

A nontrivial solution x of equation (1) is said to be oscillatory if x has arbitrarily large zeros. Otherwise, x is said to be nonoscillatory. That is, x is nonoscillatory if there exists a $t_1>t_0$ such that $x(t)\neq 0$ for $t\geq t_1$. In other words, a nonoscillatory solution must be eventually positive or eventually negative, see [**7, 9**].

2000 AMS *Mathematics Subject Classification.* Primary 34K15, 34C10.
Key words and phrases. Neutral differential equations, nonoscillatory solutions.
Research supported by Natl. Natural Sci. Foundation of P.R. China (10371103) and Research Fund of Hunan Provincial Education Dept. (04A055).
Received by the editors on Feb. 18, 2004, and in revised form on June 26, 2005.