SOME OBSERVATIONS ON GENERALIZED LIPSCHITZ FUNCTIONS

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ABSTRACT. In this paper we present a notion of generalized Lipschitz function, suggested by the Riemann-Stieltjes integral viewed as a generalization of the Riemann integral. We make some remarks on the connection of this notion with the Hellinger integral, we give a McShane's type result for generalized Lipschitz functions and a result on approximation of bounded generalized uniformly continuous function with generalized Lipschitz functions.

The classical notion of Lipschitz function is the following one:

Definition 1. Let (X, d) and (Y, d') be metric spaces. A function $f : X \to Y$ is called Lipschitz if there exists a constant $M \geq 0$ such that

$$d'(f(x), f(y)) \leq M \cdot d(x, y)$$

for all $x, y \in X$. The smallest number $M \geq 0$ satisfying the above relation is called the Lipschitz constant of f and is denoted by $\operatorname{lip} f$.

Remark 1. Intuitively speaking, a Lipschitz function is one that obeys speed limits.

From the point of view of real analysis, the condition of being Lipschitz should be viewed as a weakened version of differentiability, because of the followings result due to Rademacher, see [**6**]:

Theorem 1. *If U is an open set in $\mathbf{R}^n$ and $f : U \to \mathbf{R}^m$ is a Lipschitz function, then f is differentiable outside of a Lebesgue null subset of U.*