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CONVOLUTION OPERATORS ON SCHWARTZ SPACES FOR CHÉBLI-TRIMÈCHE HYPERGROUPS

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ABSTRACT. The convolution associated to the generalized Fourier transformation related to Chébli-Trimèche hypergroups is investigated on the Schwartz type spaces introduced by Bloom and Xu. In particular, the pointwise multipliers for these spaces are described and the convolution is studied in detail on the corresponding dual spaces.

1. Introduction. In a series of papers [**6**–**8, 23**], J.J. Betancor and Marrero studied the Hankel convolution on spaces of distributions. They developed for the Hankel convolution a theory analogous to the classical one for the usual convolution on the Schwartz distribution spaces. In this paper, we study the convolution operator associated to Chébli-Trimèche hypergroups on Schwartz type distribution spaces introduced by Bloom and Xu [**12**].

Although the notion of hypergroup was introduced in the 1930's, the harmonic analysis on hypergroups was developed in the 1970's by Dunkl [**14**], Jewett [**20**] and Spector [**26**], amongst others.

Here we deal with a special kind of hypergroup known as Chébli-Trimèche hypergroups. This sort of hypergroup has been quite investigated in the last years, see [**12, 22, 31**]. Chébli-Trimèche hypergroups are a class of one-dimensional hypergroups on $[0,\infty)$ associated to a Sturm-Liouville boundary value problem. The characters of the hypergroup are the solutions of the considered problem.

More specifically, we denote by $\triangle$ the differential operator

$$\triangle = -\frac{d^2}{dx^2} - \frac{A'(x)}{A(x)}\,\frac{d}{dx},$$

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