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A NOTE ON COMPLETENESS OF BASIC TRIGONOMETRIC SYSTEM IN $\mathcal{L}^2$

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ABSTRACT. We use the q-Lommel polynomials and the Riesz-Fisher theorem in order to give an independent proof of the completeness of the basic trigonometric system in certain weighted $\mathcal{L}^2$-spaces.

1. Introduction. In this note we present an independent proof of the completeness of the q-trigonometric system in the theory of basic Fourier series. One can look at [**2, 4, 15**–**18**] and references therein regarding the q-Fourier series. The corresponding basic exponential function on a q-quadratic grid is given by

(1.1)
$$\mathcal{E}_q(x;\alpha) = \frac{\left(\alpha^2;q^2\right)_\infty}{\left(q\alpha^2;q^2\right)_\infty} \times \sum_{n=0}^{\infty} \frac{q^{n^2/4}\ \alpha^n}{(q;q)_n}\ (-i)^n \left(-iq^{(1-n)/2}e^{i\theta}, -iq^{(1-n)/2}e^{-i\theta};q\right)_n,$$

where $x=\cos\theta$ and $|\alpha|<1$. We assume that $0<q<1$ and use the standard notations [**3**] for the basic hypergeometric series and q-shifted factorials. For the analytic continuation in a larger domain and other properties see, for example, [**8, 13, 17**] and references therein. Ismail and Zhang [**8**] found the following expansion formula

(1.2)
$$\mathcal{E}_q(x;i\omega) = \frac{(q;q)_\infty\, \omega^{-\nu}}{(q^\nu;q)_\infty\left(-q\omega^2;q^2\right)_\infty} \times \sum_{m=0}^{\infty} i^m \left(1-q^{\nu+m}\right)\ q^{m^2/4} J^{(2)}_{\nu+m}(2\omega;q)\ C_m\left(x;q^\nu \mid q\right),$$

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