ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 33, Number 4, Winter 2003

ESSENTIAL NORMS OF WEIGHTED COMPOSITION OPERATORS BETWEEN BLOCH-TYPE SPACES

BARBARA D. MACCLUER AND RUHAN ZHAO

ABSTRACT. We compute the essential norm of a weighted composition operator uC_φ acting from an analytic Lipschitz space into a weighted Bloch-type space on the disk, and give estimates for the essential norm of uC_φ when it maps the standard Bloch space into a weighted Bloch-type space. We also study boundedness and compactness of weighted composition operators on analytic Lipschitz spaces from a geometric perspective.

1. Introduction. Let D be the open unit disk in the complex plane. Let u be a fixed analytic function on D and φ an analytic self-map of D. We can define a linear operator uC_φ on the space of analytic functions on D, called a *weighted composition operator*, by

$$uC_\varphi f = u(f \circ \varphi).$$

It is easy to see that an operator defined in this manner is linear. We can regard this operator as a generalization of a multiplication operator and a composition operator.

Our main interest here is in determining the essential norm of a weighted composition operator acting from one weighted Bloch space, defined below, to another. In the case that $u(z) \equiv 1$ or $\varphi(z) = z$, our formulas give the essential norm of the multiplication operator M_u or the composition operator C_φ, respectively.

Essential norm formulas for composition operators are known in various settings. When C_φ acts from the Hardy space $H^2(D)$ to itself, Shapiro [**9**] gives a formula for $\|C_\varphi\|_e$, the essential norm of C_φ, in terms of the Nevanlinna counting function for φ. A similar formula, using a generalized Nevanlinna counting function, for the essential norm of C_φ acting on the Bergman space $A^2(D)$ is given in [**6**]. In [**3**], Donaway

Received by the editors on August 7, 2001, and in revised form on November 6, 2001.