INVERTIBILITY CONDITIONS FOR BLOCK MATRICES AND ESTIMATES FOR NORMS OF INVERSE MATRICES

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ABSTRACT. A nonsingularity criterion for block matrices is derived. It improves the well-known results in the case of matrices which are "close" to block triangular ones. Moreover, an estimate for the norm of the inverse matrices is derived.

1. Introduction and statement of the main result. Although excellent computer software are now available for eigenvalue computation, new results on invertibility and spectrum inclusion regions for finite matrices are still important, since computers are not very useful, in particular, for analysis of matrices dependent on parameters. But such matrices play an essential role in various applications, for example, in stability and boundedness of coupled systems of partial differential equations, cf. [**9**, Section 14]. In addition, invertibility conditions for finite matrices allow us to derive invertibility conditions for linear operators and, in particular, infinite matrices. Because of this, the problem of finding invertibility conditions and spectrum inclusion regions for finite matrices continues to attract the attention of many specialists, cf. [**1–3, 8, 10–12, 14**] and references given therein.

Many books and papers are devoted to the invertibility of block matrices [**4, 5, 6, 13**], etc. In these works mainly, the Hadamard theorem is generalized to block matrices. Note that the generalized Hadamard theorem does not assert that a block triangular matrix with nonsingular diagonal blocks is invertible. But it is not hard to check that such a matrix is always invertible. In the present paper, we propose invertibility conditions which improve well-known results for matrices that are "close" to block triangular matrices. Moreover, we derive an estimate for the norm of the inverse matrices.