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COHOMOLOGICAL PROPERTIES OF MULTIPLE COVERINGS OF SMOOTH PROJECTIVE CURVES

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ABSTRACT. Let X, respectively C, be a smooth projective curve of genus g, respectively q, and $f : X \to C$ a degree k finite morphism. Set $E := f_*(\mathbf{O}_X)/\mathbf{O}_C$. Hence E is a rank $k-1$ vector bundle on C with $\deg(E) = kq - k + 1 - g$. Here we study the cohomological properties of E and in particular the integers $h^0(C, E(tP))$, $P \in C$ and $t \in \mathbf{N}$. We use these integers to define the notion of f-Weierstrass points.

1. Introduction. Let X, respectively C, be a smooth connected projective curve of genus g, respectively q, defined over an algebraically closed base field $\mathbf{K}$ and $f : X \to C$ a degree k covering. Set $E := f_*(\mathbf{O}_X)/\mathbf{O}_C$. The sheaf E is a rank $k-1$ vector bundle on C. We will say that E is the bundle associated to f. Many geometrical properties of X are detected by the cohomological properties of E. If $q = 0$, then E is a direct sum of line bundles and the degrees of the rank 1 summands of E uniquely determine the so-called scrollar invariants of the pencil f (see [**12**, Section 2]). In this paper we will consider the case $q > 0$. If either $\mathrm{char}\,(\mathbf{K}) = 0$ or $\mathrm{char}\,(\mathbf{K}) > k$, the trace map induces a splitting $f_*(\mathbf{O}_X) \cong \mathbf{O}_C \oplus E$; since X is connected, in this case we have $h^0(C, E) = 0$. For any $P \in C$ and any integer t, set $n(f, P, t) := h^0(C, E(tP))$ and $N(f, P, t) := h^0(C, f_*(\mathbf{O}_X)(tP)) = h^0(X, (f^{-1}(P))^{\otimes t})$ (projection formula). The sequence $n(f, P) := \{n(f, P, t)\}_{t \geq 0}$, respectively $N(F, P) := \{N(f, P, t)\}_{t \geq 0}$, will be called the numerical sequence, respectively the total numerical sequence, of f at P. Set $n(f, t) := n(f, P, t)$ and $N(f, t) := N(f, P, t)$ for general $P \in C$. The sequence $n(f) := \{n(f, t)\}_{t \geq 0}$, respectively $N(f) := \{N(f, t)\}_{t \geq 0}$, will be called the numerical sequence, respectively the total numerical sequence, of f. If $P \in C$ and $n(f, P, t) \neq f(f, t)$ for

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