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# AN ASYMPTOTIC PROPERTY FOR TAILS OF LIMIT PERIODIC CONTINUED FRACTIONS

LISA JACOBSEN AND HAAKON WAADELAND

**1. Introduction.** In the present paper we study continued fractions of the forms

$$K_{n=1}^{\infty}\frac{a_n}{1} = K(a_n/1) = \frac{a_1}{1} {}_{+} \frac{a_2}{1} {}_{+} \cdots {}_{+} \frac{a_n}{1} {}_{+} \cdots, \tag{1.1}$$

$$a_n \in \mathbf{C}\setminus\{0\},$$

and

$$K_{n=1}^{\infty}\frac{1}{b_n} = K(1/b_n) = \frac{1}{b_1} {}_{+} \frac{1}{b_2} {}_{+} \cdots {}_{+} \frac{1}{b_n} {}_{+} \cdots, \tag{1.2}$$

$$b_n \in \mathbf{C},$$

where the elements $\{a_n\}$, $\{b_n\}$ are *limit $k$-periodic* for a $k \in \mathbf{N}$, that is

$$a_{kn+p} = \tilde{a}_p + \delta_{kn+p} \text{ or } b_{kn+p} = \tilde{b}_p + \delta_{kn+p};\ \tilde{a}_p, \tilde{b}_p \in \mathbf{C}, \delta_n \to 0 \tag{1.3}$$

for $p = 1, \ldots, k$ and all $n \geq 0$. We also assume that these limit periodic continued fractions are of hyperbolic or loxodromic type. (For definition, see §2 and §3.) It is then well known that the continued fraction converges, at least generally. (For definition, see §3.) So do also all its *tails*

$$\begin{aligned} K_{n=1}^{\infty}\frac{a_{m+n}}{1} &= \frac{a_{m+1}}{1} {}_{+} \frac{a_{m+2}}{1} {}_{+} \cdots \\ K_{n=1}^{\infty}\frac{1}{b_{m+n}} &= \frac{1}{b_{m+1}} {}_{+} \frac{1}{b_{m+2}} {}_{+} \cdots \end{aligned} \tag{1.4}$$

for $m \in \mathbf{N}\cup\{0\}$. Let $f^{(m)}$ denote the value of the $m$-th tail (1.4). Then $\{f^{(m)}\}$ is also limit $k$-periodic [**3**, p. 96; **1**].

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