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A NOTE ON THE CYCLIC COHOMOLOGY AND K-THEORY ASSOCIATED WITH DIFFERENCE OPERATORS

DAOXING XIA

ABSTRACT. The index map of K_0-theory associated with a difference operator is given. In the odd dimension case, a theorem on the cyclic cohomology is established.

1. This note is a continuation of the author's previous paper [**2**]. Let $\mathcal{A}$ and $\mathcal{A}_1$ be two algebras over $\mathbf{C}$ satisfying $\mathcal{A} \subset \mathcal{A}_1$. As it is introduced in [**2**], an operator δ from $\mathcal{A}$ into $\mathcal{A}_1$ is said to be a difference operator if δ is linear and satisfies

$$\delta(fg) = f\delta g + (\delta f)g - (\delta f)\delta g \tag{1}$$

for $f, g \in \mathcal{A}$.

In [**2**], the following theorem is proved.

THEOREM. *Let $\mathcal{H}$ be a Hilbert space, $\mathcal{A}$ a subalgebra of $\mathcal{L}(\mathcal{H})$ and δ a difference operator from $\mathcal{A}$ into $\mathcal{L}(\mathcal{H})$ satisfying*

$$\delta f \in \mathcal{L}^p(\mathcal{H}), \quad f \in \mathcal{A},$$

where $p \geq 1$. Let n be an even number satisfying $n \geq p-1$ and

$$\psi_n(f_0, \dots, f_n) = \operatorname{tr}(\delta f_0 \cdots \delta f_n), \quad f_0, \dots, f_n \in \mathcal{A}.$$

Then ψ_n is a cyclic cocycle. If $n \geq p+1$, then ψ_n is in the cyclic cohomology class containing $bR_{n-2}\psi_{n-2}$, where R_k is the operation

$$(R_k\xi)(f_0, f_1, \dots, f_{k+1}) = \frac{2}{k+2}\sum_{j=0}^{k}(-1)^j(k-j+1)\xi(f_j f_{j+1}, f_{j+2}, \dots, f_{j+k+1})$$

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