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THE BOUNDEDNESS CONDITION OF DILATION THEORY CHARACTERIZES SUBNORMALS AND CONTRACTIONS

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TO THE MEMORY OF CONSTANTIN APOSTOL

1. Introduction. At the foundations of the general dilation theory on semigroups there are two conditions: positive definiteness PD and the boundedness condition BC /see definitions below/. In general, PD is considered to be more basic than BC, essentially because of the traditional and the most natural method of constructing the dilation Hilbert space by introducing the associated sesquilinear form, positivity of which is guaranteed by PD. The core of this method goes back to classical works of Kolmogoroff, Moore-Aronszajn, Krein, Koranyi-Sz.-Nagy, and others—see [**6**, KMKA Lemma] for references. An abstract version of this method can be found in [**11**], where it is also shown that, assuming PD, dilations can be constructed under conditions much weaker than BC, but these dilations are far from being bounded, even if semigroups in question have involutions. BC can be seen, in general, as the condition that guarantees boundedness of dilations. This general approach applies to a single operator theory in two important cases: unitary dilations of contractions and normal extensions of subnormal operators, which has been done by Sz.-Nagy [**9, 8**], following, for subnormals, Halmos's positivity condition [**2**].

In both cases BC is a consequence of PD. For a single contraction the associated PD function is defined on the group (of integers), which makes BC disappear. That, for a subnormal operator, the associated PD function satisfies BC, was proved by Bram [**1**] who used a deep result of Heinz [**3**]. Szafraniec [**7**] was able to show this without Heinz's result, but applying instead his remarkably simplified BC for $*$-semigroups, which is a consequence of a very careful and elaborate use of Schwarz's inequality. These problems for semigroups of contractions and subnormal semigroups are discussed in [**4**, **10**] and [**12**], respectively. Therefore it seems that BC is insignificant in these

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