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DIRECT INTEGRALS OF STANDARD FORMS OF W^*-ALGEBRAS

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ABSTRACT. Bös [Invent. Math. **37** (1976), p. 241] proved that standard forms of W^*-algebras behave naturally with respect to direct integrals. We give a new approach to disintegration of standard forms, which uses the characterization of matrix-ordered Hilbert spaces in standard forms of W^*-algebras obtained by Wittstock and the author [Math. Scand. **51** (1982), p. 241].

Introduction. Araki [**1**], Connes [**3**] and Haagerup [**7**] developed standard forms of W^*-algebras. Connes [**3**] characterized the ordered Hilbert spaces arising in these standard forms. Penney [**8**] developed direct integrals of selfdual cones. Based on [**3**], and [**8**], Bös showed in [**2**] that standard forms behave naturally with respect to direct integrals. Wittstock and the author [**11, 12**] characterized the Hilbert spaces arising in standard forms of W^*-algebras among matrix ordered spaces. In this note we give a self contained and simplified approach to disintegration of standard forms. In fact proper use of a result of Elliott [**5**] makes it possible to work with only a few consequences of the measurable choice theorem due to Sainte-Beuve [**9**]. Furthermore disintegration of matrix order allows us to dispense with the rather technical direct integral of orientations [**2**] and is therefore more natural from a categorial point of view.

1. Technical preliminaries.

1.1 *Separability conditions.*

PROPOSITION. *Let $\mathcal{M}$ be a W^*-algebra. Then the following conditions are equivalent*:

a) *$\mathcal{M}$ has a separable predual $\mathcal{M}_*$.*

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