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ON EXACT CONTROLLABILITY OF OPERATORS

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Dedicated to the memory of Constantin Apostol

1. Introduction. Let $\mathcal{X}, \mathcal{Y}$ be Banach spaces, and let $A \in L(\mathcal{Y}), B \in L(\mathcal{X}, \mathcal{Y})$ (as usual, $L(\mathcal{X}, \mathcal{Y})$ stands for the Banach space of all linear bounded operators from $\mathcal{X}$ into $\mathcal{Y}$, and $L(\mathcal{Y})$ is the abbreviation of $L(\mathcal{Y}, \mathcal{Y})$). The pair (A, B) is called *exactly controllable* if

$$\mathcal{Y} = \bigcup_{n=1}^{\infty} \Big(\sum_{j=0}^{n-1} \operatorname{Im} A^{n-1-j} B \Big).$$

(Here and elsewhere we denote

$$\operatorname{Im} S = \{ Sx \,|\, x \in \mathcal{X} \}$$

for $S \in L(\mathcal{X}, \mathcal{Y})$.) This notion appears naturally in linear systems theory (an indication to that is given in the next section) and was studied by several authors [**1, 2, 3, 6, 8**]. In the finite dimensional case ($\dim \mathcal{Y} < \infty$) the notion of exact controllability is one of the most important in modern linear system theory and can be found in virtually every book on the subject (see, e.g., [**5, 10**]).

A crucial property of exactly controllable pairs in the finite dimensional case is the following fact (known as the pole, or spectrum, assignment theorem, see [**5, 9**]): A pair (A, B) is exactly controllable if and only if, for every m-tuple (here $m = \dim \mathcal{Y}$) of complex numbers $\lambda_1, \dots, \lambda_m$ there is $F \in L(\mathcal{Y}, \mathcal{X})$ such that $\sigma(A+BF) = \{\lambda_1, \dots, \lambda_m\}$. Recently, infinite dimensional versions of these results were proved in [**2, 8**].

In this paper we make more precise the spectrum assignment results for exactly controllable pairs proved in [**8**] by exhibiting the continuous dependence on the parameters involved.

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