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BIMODULES OVER CARTAN SUBALGEBRAS

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ABSTRACT. Given a Cartan subalgebra $\mathbf{A}$ of a von Neumann algebra $\mathbf{M}$, the techniques of Feldman and Moore are used to analyze the partial isometries v in $\mathbf{M}$ such that $v^*\mathbf{A}v$ is contained in $\mathbf{A}$. Orthonormal bases for $\mathbf{M}$ consisting of such partial isometries are discussed, and convergence of the resulting generalized fourier series is shown to take place in the Bures $\mathbf{A}$-topology. The Bures $\mathbf{A}$-topology is shown to be equivalent to the strong topology on the unit ball of $\mathbf{M}$. These ideas are applied to $\mathbf{A}$-bimodules in $\mathbf{M}$ to prove the existence of orthonormal bases for bimodules and to give a simplified and intuitive proof of the Spectral Theorem for Bimodules first proven by Muhly, Saito, and Solel.

1. Introduction. The notion of a *triangular subalgebra* of a von Neumann algebra $\mathbf{M}$ was introduced in a paper of Kadison and Singer [**7**]; it is defined to be an algebra $\mathbf{T}$ of operators in $\mathbf{M}$ such that $\mathbf{T} \cap \mathbf{T}^*$ is a maximal abelian subalgebra $\mathbf{A}$ of $\mathbf{M}$, called the *diagonal* of $\mathbf{T}$. In a recent paper of Muhly, Saito and Solel [**8**] (referred to here as MSS) triangular subalgebras whose diagonal is a Cartan subalgebra of $\mathbf{M}$ were considered and analyzed using a formalism developed by Feldman and Moore in [**4, 5**] which is summarized below. A subspace of $\mathbf{M}$ which is invariant under left and right multiplication by members of $\mathbf{A}$ is called an $\mathbf{A}$-*bimodule*; this class of subspaces includes subalgebras of $\mathbf{M}$ containing $\mathbf{A}$ and in particular triangular subalgebras with diagonal $\mathbf{A}$. In MSS a critical role was played by the Spectral Theorem for Bimodules in which σ-weakly closed $\mathbf{A}$-bimodules are characterized.

In the introduction of MSS a simple and elegant motivation of the Spectral Theorem for Bimodules is given in the finite-dimensional case, but the proof of this theorem in the context of a Cartan subalgebra doesn't follow the motivation given. In §5 of this paper it is shown that the motivation and proof of the finite dimensional case can in fact be carried through to the case of a Cartan subalgebra. Some preliminary

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