ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 20, Number 2, Spring 1990

ON THE EQUIVALENCE OF THE OPERATOR EQUATIONS $XA + BX = C$ AND $X - p(-B)Xp(A)^{-1} = W$ IN A HILBERT SPACE, p A POLYNOMIAL

TAPAS MAZUMDAR AND DAVID F. MILLER

ABSTRACT. We consider the solution of $(*)$ $XA + BX = C$ for bounded operators A, B, C and X on a Hilbert space, A normal. We establish the existence of a polynomial p and a bounded operator W with the property that the unique solution X of $(*)$ also solves $X - p(-B)Xp(A)^{-1} = W$ uniquely. A known iterative algorithm can be applied to the latter equation to solve $(*)$.

1. Introduction and notations. We know that the equation

$$XA + BX = C \tag{1}$$

with

$$\sigma(A) \cap \sigma(-B) = \phi \tag{2}$$

in which A, B, C are given finite-dimensional matrices of compatible orders, and $\sigma(T)$ is the spectrum of the matrix T (or possibly the operator T), has a unique matrix solution X [**7,9**]. Letting $r(T)$ denote the spectral radius of T, an iterative method to calculate the matrix solution X of the system (1), (2) is obtained *if* we can rewrite (1) in an equivalent form

$$X - UXV = W \tag{3}$$

with

$$r(U)r(V) < 1. \tag{4}$$

When this is possible, the recursion

$$X_{k+1} = U^{2^k} X_k V^{2^k} + X_k, \qquad X_0 = W \tag{5}$$

Received by the editors on November 6, 1987 and, in revised form on March 21, 1988.