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INVARIANT SUBSPACES AND THIN SETS

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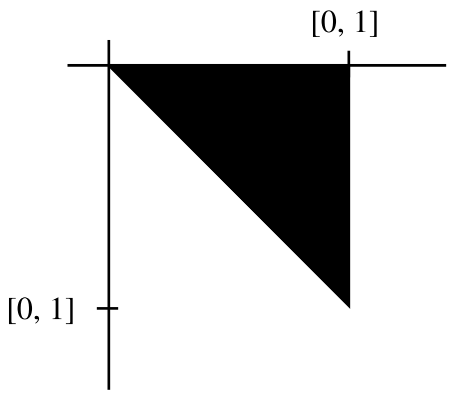
This expository article will outline some connections between the existence of compact operators in reflexive operator algebras with a commutative subspace lattice (CSL algebras) and the theory of "thin sets" in harmonic analysis. Full details will appear elsewhere [**5**].

Let X be a compact metric space, μ a finite Borel measure on X and $\leq$ a closed partial-order on X. The operator algebra $\mathrm{Alg}\,(X, \leq, \mu)$ is described in [**1**] where its main properties are developed. We mention that

$$\mathrm{Lat}\,(\mathrm{Alg}\,(X, \leq, \mu) = \mathcal{L}\,(X, \leq) = \{P_E : E \text{ is a decreasing Borel set}\}.$$

We are concerned with the existence of compact operators in $\mathrm{Alg}\,(X, \leq, \mu)$.

EXAMPLE 1. Let $X = [0, 1]$, with Lebesgue measure dx and the usual linear order. Then $A = \mathrm{Alg}\,([0, 1], \leq, dx)$ is a nest algebra consisting of all operators on $L^2[0, 1]$ "supported" on the graph of the linear order



$$\mathrm{Lat}\,(A) = \{P_{[0,r]} : 0 \leq r \leq 1\}.$$

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