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HYPONORMAL AND QUASINORMAL WEIGHTED COMPOSITION OPERATORS ON ℓ^2

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ABSTRACT. Characterizations of hyponormal and quasinormal weighted composition operators on the Hilbert space of complex valued functions on the integers are given in this article. Similar results are also presented for the adjoints of weighted composition operators.

Let H be a Hilbert space of complex-valued functions defined on a set X. A *weighted composition operator* on H is usually defined by $Tf = u(f \circ g)$ for all f in H where $u : X \to \mathbf{C}$ is a *weight function* and $g : X \to X$ is a *composition function.* An operator A on H is *hyponormal* if $A^*A - AA^* \geq 0$, is *seminormal* if one of A or A^* is hyponormal, and is *quasinormal* if $AA^*A = A^*AA$. In the very general setting where H is a sigma-finite L^2 space, measure theoretic characterizations of hyponormal weighted composition operators have been obtained by A. Lambert [**3**] and measure theoretic characterizations of hyponormal, seminormal, and quasinormal unweighted composition operators have been obtained by D. Harrington and R. Whitley [**2**].

In this article we will restrict ourselves to the Hilbert space ℓ^2 of complex-valued functions on the integers ($\mathbf{Z}$ = integers, $\mathbf{C}$ = complex numbers). We will also generalize the definition of a weighted composition operator: for y a subset of $\mathbf{Z}$, $g : y \to \mathbf{Z}$ and $u : \mathbf{Z} \to \mathbf{C} \setminus \{0\}$, define the weighted composition operator T_{ug} by

$$T_{ug}f(n) = \begin{cases} u(n) f \circ g(n) & \text{for } n \text{ in } y, \\ 0, & \text{for } n \text{ not in } y. \end{cases}$$

Characterizations of when T_{ug} and T_{ug}^* are hyponormal, seminormal and quasinormal are presented here. The characterization of T_{ug} hyponormal is a more concrete example of the characterization given in Lambert [**3**]. Here we have slightly generalized the definition of T_{ug} to encompass more operators on ℓ^2 and we have given an independent

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