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ON PERTURBATIONS OF REFLEXIVE ALGEBRAS

HARI BERCOVICI AND FLORIN POP

We denote by $\mathcal{H}$, $\mathcal{L}(\mathcal{H})$, and $\mathcal{K}$ a complex Hilbert space, the algebra of bounded linear operators on $\mathcal{H}$, and the ideal of compact operators on $\mathcal{H}$, respectively. We recall that a subalgebra $\mathcal{A} \subset \mathcal{L}(\mathcal{H})$ is said to be *reflexive* if it contains every operator T such that $T\mathcal{M} \subset \mathcal{M}$ whenever $\mathcal{M}$ is closed invariant subspace for $\mathcal{A}$.

In this paper we provide elementary examples that answer in the negative the following two questions.

PROBLEM 1. Suppose that $\mathcal{A} \subset \mathcal{L}(\mathcal{H})$ is a reflexive algebra. Is then $\mathcal{A} + \mathcal{K}$ norm-closed?

PROBLEM 2. Suppose that $\mathcal{A}_n$, $\mathcal{A} \subset \mathcal{L}(\mathcal{H})$ are similar reflexive algebras, $n \geq 0$, and $\lim_{n\to\infty} \operatorname{dist}(\mathcal{A}_n, \mathcal{A}) = 0$. Can we choose invertible operators X_n such that $X_n^{-1}\mathcal{A}X_n = \mathcal{A}_n$ and $\lim_{n\to\infty} \|X_n - I\| = 0$?

The distance mentioned in Problem 2 is, of course, the Pompeiu-Hausdorff distance between the unit balls of $\mathcal{A}_n$ and $\mathcal{A}$.

We note that Problem 1 has an affirmative answer if the invariant subspaces of $\mathcal{A}$ are totally ordered by inclusion (i.e., $\mathcal{A}$ is a nest algebra); see [**6**]. The answer to Problem 1 is negative for algebras with commutative invariant subspace lattice (CSL-algebras); see [**7**]. See also [**1**] and [**11**] for more details about such algebras.

The answer to Problem 2 is positive if $\mathcal{A}_n$ and $\mathcal{A}$ are nest algebras. Problem 2 has a negative answer is $\mathcal{A}$ is a CSL-algebra (see [**5**]), but it is open for algebras acting on finite-dimensional spaces. See [**2, 3, 4, 10** and **12**] for more information about this problem.

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