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THE AVERAGE ERROR OF QUADRATURE FORMULAS FOR FUNCTIONS OF BOUNDED VARIATION

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1. Introduction. The purpose of this paper is to discuss the average case error of quadrature formulas for functions of bounded variation. If one wants to consider average case errors then the main problem is to define a natural probability measure on the function set in question. One natural class of such measures is that of the Gaussian measures which have been considered by several authors (see for instance [**6, 2, 7, 3**]).

We take an alternative approach. The probability measures we are interested in should reflect "uniform distribution" in some sense (see also [**4**]). On bounded sets of finite dimensional spaces the normalized Lebesgue measure is the canonical candidate. On infinite dimensional spaces a translation invariant measure which is finite on bounded sets does not exist. Therefore, we construct a probability measure Q on the set $BV = \{f : [0,1] \to \mathbf{R} \,|\, f \text{ continuous}, \ f(0) = 0, \mathrm{Var}\,(f) \leq 1\}$ in a different way using the "natural" measure on the homeomorphisms of $[0,1]$ introduced in [**1**].

Let e_n^Q denote the infimum of the average errors of quadrature formulas with n knots. We show that e_n^Q converges to 0 like $n^{-\log 6/(2\log 2)}$, where $\log 6/(2\log 2) = 1.29248\ldots$. This contrasts with the result for the worst case analysis. Much as in [**8**] one can show that, among all quadrature formulas with n knots, the rule

$$f \to \frac{1}{n}\sum_{i=1}^{n} f\Big(\frac{2i}{2n+1}\Big)$$

has minimal maximal error $1/(2n+1)$.

2. A probability measure Q on BV. Let H be the space of all homeomorphisms h from $[0,1]$ onto itself with $h(0) = 0$ and $h(1) = 1$ equipped with the topology of uniform convergence.

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