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PERIODIC GENERALIZED FUNCTIONS

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ABSTRACT. A class of periodic generalized functions, called periodic Boehmians, is studied. Each periodic Boehmian is the sum of its Fourier series. The class of periodic Boehmians is strictly smaller than the class of periodic Mikusiński operators, and strictly larger than the class of periodic distributions.

1. Introduction. In this paper we shall construct the Boehmians on the unit circle. For a general construction of Boehmians see [**6**].

Generalized functions on the unit circle have been classified by their Fourier coefficients. For example, $\{\alpha_n\}_{-\infty}^{\infty}$ is the sequence of Fourier coefficients of a distribution if the α_n's grow no faster than a polynomial in n [**7**]. $\{\alpha_n\}_{-\infty}^{\infty}$ is the sequence of Fourier coefficients of a hyperfunction if $\overline{\lim}_{|n|\to\infty} |\alpha_n|^{1/|n|} \le 1$ [**4**]. Any sequence of complex numbers is the sequence of Fourier coefficients of a Mikusiński operator [**3**]. We will show that the coefficients of a periodic Boehmian satisfy a growth condition much like that of a hyperfunction.

§2 is concerned with definitions. Most of the material in §3 and §4 can be found in [**6**] and [**2**], respectively, but is presented here for the convenience of the reader. §3 has results on convergence. §4 gives an example of a periodic Boehmian which is not a distribution. In §5 Fourier coefficients are defined and it is shown that the Fourier coefficients of a periodic Boehmian satisfy a growth condition (Theorem 5.14 and Theorem 5.15). It is not known whether the condition in Theorem 5.14 is necessary and sufficient. Indeed there is a significant gap between the condition in Theorem 5.14 and the condition in Theorem 5.15; it is not even known if each sequence which is $o(e^{o(n)})$ is a sequence of Fourier coefficients for a periodic Boehmian.

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