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NORMALITY FOR THE PROBLEM OF BOLZA WITH AN INEQUALITY STATE CONSTRAINT

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1. Introduction and preliminary results. The existence of a strong relation between the normality assumption in optimal control and the controllability of the associated variational equation has long since been noted. Here we extend the previous results to the problem of Bolza with general endpoint conditions and in the presence of a state inequality constraint. While the restriction on the initial and final point becomes, through linearization, a linear boundary condition on the variational equation, the state constraint, which induces a forcing term in the adjoint equation, becomes an isoperimetric condition. This kind of correspondence between linear differential equations and its *adjoint* has been introduced in [**5**] but, for the optimal control problem considered here, is new and requires further investigation.

In this paper we consider the problem of minimizing the cost functional

$$J(x,u) = g(x(a),x(b)) + \int_a^b f_0(s,x(s),u(s))\,ds \tag{1.1}$$

over all absolutely continuous functions, $x(\cdot)$, and measurable functions, $u(\cdot)$, satisfying

$$\begin{gathered} \dot{x}(t) = f(t,x(t),u(t)), \quad \text{a.e. } t \in I, \\ u(t) \in U, \quad \text{a.e. } t \in I, \\ \psi(x(a),x(b)) = 0, \\ \phi(x(t)) \leq 0, \quad t \in I, \end{gathered} \tag{1.2}$$

where, for given open sets $X \subset \mathbf{R}^n$ and $V \subset \mathbf{R}^m$, $U \subset V$, and for

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