ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 20, Number 4, Fall 1990

INVASION OF A PERSISTENT SYSTEM

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Dedicated to the memory of Geoffrey J. Butler, my doctoral thesis supervisor. His influence and inspiration continue to live on.

1. Introduction. Today, with genetic engineering no longer just a topic of science fiction, but rather a reality, one of the intriguing questions in ecology concerns how to predict the effect of introducing a new species to a thriving ecosystem. In this paper we consider the following special case: When is it possible for an invading population to successfully infiltrate a community?

We formulate the problem in terms of the mathematical notions of *persistence* (see, for example, Butler, Freedman, and Waltman [**2, 3**] and Butler and Waltman [**4**]). Other closely related terminology includes *cooperativity, permanent coexistence, permanence*, and *ecological stability.* For a discussion of how these terms are related, see Gard [**8**], Hofbauer [**10**] and Hutson and Law [**14**]. The notion of *uninvadability* is discussed in Sigmund and Schuster [**16**].

2. Preliminaries. For any positive integer n, define

$$\mathbf{R}_+^n = \{(x_1, x_2, \dots , x_n) \in \mathbf{R}^n : x_i \geq 0,\ i = 1, \dots , n\}$$

and, for any $J \subseteq \mathcal{N} = \{1, 2, \dots , n\}$, define

$$\mathbf{R}_J^n = \{(x_1, x_2, \dots , x_n) \in \mathbf{R}_+^n : x_i = 0,\ i \in \mathcal{N}\backslash J\}.$$

Consider the autonomous system

$$\begin{gathered} \dot{z}(t) = g(z(t)) \quad (z = (z_1, z_2, \dots , z_k)), \\ z(0) \in \operatorname{int} \mathbf{R}_+^k \quad \left(\dot{} = \frac{d}{dt}\right), \end{gathered} \tag{2.1}$$

Research partially supported by Natural Science and Engineering Research Council of Canada grant #NSERC A-9358 and by a Science and Engineering Research Board grant from McMaster University.