ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 20, Number 4, Fall 1990

NODAL OSCILLATION AND WEAK OSCILLATION OF ELLIPTIC EQUATIONS OF ORDER 2m

V. B. HEADLEY

1. Introduction. Let L and M_0 be differential operators defined by

$$Lu = \sum_{|\alpha|=0}^{m} \sum_{|\beta|=0}^{m} (-1)^{|\alpha|} D^\alpha [A_{\alpha\beta}(x) D^\beta u], \quad x \in \Omega \subseteq \mathbf{R}^n, \tag{1.1}$$

and

$$M_0 v = (-1)^m \sum_{|\alpha|=|\beta|=m} D^\alpha [a_{\alpha\beta}(x) D^\beta v] + a_0(x) v, \tag{1.2}$$

where the coefficient functions $A_{\alpha\beta}$ and $a_{\alpha\beta}$, $|\alpha| \le m$, $|\beta| \le m$, are real-valued, satisfy the symmetry conditions

$$A_{\alpha\beta} = A_{\beta\alpha}(x), \quad x \in \Omega,\ |\alpha| \le m,\ |\beta| \le m, \tag{1.3}$$

$$a_{\alpha\beta}(x) = a_{\beta\alpha}(x), \quad |\alpha| = |\beta| = m,\ x \in \Omega, \tag{1.4}$$

and are sufficiently smooth on the unbounded open set Ω. (The multi-index notation employed here is that used in [**1, 2** and **6**].) In this paper the sign of $a_0(x)$ is *unrestricted*, unless the contrary is stated.

HYPOTHESIS 1.1. Throughout this paper, G will denote a nonempty open subset of Ω. (We will occasionally need to consider the special case where $G = \Omega$.)

DEFINITION 1.2. If G is bounded and satisfies the hypotheses of [**2**, Lemma 9.1], and if the differential equation

$$Lu = 0 \tag{1.5}$$

has a nontrivial solution u in $H_m^0(G) \cap C^{2m}(G)$, then G is called a *nodal domain* for L. We will say that (1.5) is *nodally oscillatory* in Ω iff, for