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ON THE STABILITY OF ONE-PREDATOR TWO-PREY SYSTEMS

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1. Introduction. The MacArthur-Rosenzweig "graphical criterion" of stability says, loosely speaking, that if, in a predator-prey system, the interior equilibrium point lies on the decreasing branch of the prey's zero-isocline, then it is asymptotically stable; it if lies on the increasing branch (in the prey-predator phase plane) then it *may be* unstable (see [**7, 3**]; in case the predator's zero-isocline is a vertical straight line i.e., there is no intraspecific competition in the predator species, it *is* unstable). Freedman and the author have generalized this criterion to the three-dimensional case when there are two predator species competing for a single prey species [**1, 2**]. We have shown that if there is no direct interspecific competition between the predator species and the derivative with respect to the prey quantity of the specific growth rate function of the prey is negative at the interior equilibrium, then this equilibrium is asymptotically stable. In [**2**] we have shown by some drawings the intuitive geometric meaning of the MacArthur-Rosenzweig criterion, namely, that if the condition is fulfilled, and the system is driven out of the equilibrium in an easily controllable way, then the dynamics drives it closer to the equilibrium.

In the present paper we are going to show that the MacArthur-Rosenzweig criterion does not generalize to three-dimensional systems with two competing prey species in the general case. We are giving sufficient conditions for the asymptotic stability of an interior equilibrium. The conditions might be considered more or less known, at least in case the specific growth rates are linear functions, i.e., in case we have a Lotka-Volterra system (see Hutson and Vickers [**5**] and the references therein and Svirezhev, Logofet [**8**]). The relation of these conditions to some concerning permanent coexistence will also be pointed out (cf: [**5**] and see also [**4, 6**]). We shall show the special case in which the MacArthur-Rosenzweig criterion can be generalized.

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