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SOLVABILITY OF TWO-POINT BOUNDARY VALUE PROBLEMS FOR SYSTEMS OF NONLINEAR DIFFERENTIAL EQUATIONS OF THE FORM $\mathbf{y}'' = \mathbf{g}(\mathbf{t}, \mathbf{y}, \mathbf{y}', \mathbf{y}'')$

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Introduction. Two authors of this paper have proved in [**7**] a number of existence results for systems of differential inclusions

$$y'' \in F(t, y, y'), \quad a.e.\ t \in I, \tag{1}$$

subject to various boundary conditions. In the above, I stands for either a finite interval or a half-line, F is an "admissible" convex-valued multifunction, $y \in C^1(I; \mathbf{R}^n)$ and $y'' \in L^2(I; \mathbf{R}^n)$ ($y'' \in L^2_{\text{loc}}(I; \mathbf{R}^n)$ in case $I = [0, \infty)$.

With the help of those results, we establish the solvability of the same boundary value problems for systems of differential equations

$$y'' = g(t, y, y', y''), \quad a.e.\ t \in I, \tag{2}$$

where g is a Caratheodory function which, loosely speaking, satisfies Bernstein-Nagumo-type conditions, with respect to (y, y') and is nonexpansive in y''. Unlike the contraction principle, fixed point theorems for nonexpansive maps do not guarantee the unique solvability of $y'' = g(t, y, y', y'')$ with respect to y'', so we cannot reduce (2) to the classical equation

$$y'' = F(t, y, y'), \quad a.e.\ t \in I,$$

with $F(t, y, y')$ being the fixed point. The set of fixed points of a nonexpansive mapping is, however, convex, and, by denoting it by $F(t, y, y')$, we reduce (2) to (1). In Section 1, we prove that this "implicit multifunction" $F(t, y, y')$ has the required properties so that the results of [**7**] may be applied. In Section 2 we derive from those results analogous conclusions about boundary value problems for (2).

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