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ITERATION OF MÖBIUS TRANSFORMS AND CONTINUED FRACTIONS

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Dedicated to Professor W.J. Thron
on the occasion of his 70th birthday.

1. Introduction. In this paper we study sequences of complex points obtained by successive application of Möbius transforms to a starting point. Lately a large literature dealing with iteration of simple maps, notably quadratic polynomials, has appeared ([2] and references therein). It might seem to be of interest to do similar studies for Möbius transforms. However, the dynamics of Möbius transforms is much simpler (cf. Section 2), largely due to the fact that composition of two Möbius transforms is again a Möbius transform, while the composition of a quadratic polynomial with itself is a fourth degree polynomial. This latter fact is crucial for the period doubling scenario of the quadratic maps.

A good reason for studying the effect of repeated applications of Möbius transforms is that it relates to continued fractions. To be a bit more precise, let s_k be Möbius transforms of the special type $s_k(z) = a_k/(1+z)$; then the *convergents* of the continued fraction

$$\underset{k=1}{\overset{\infty}{K}}(a_k/1) = \cfrac{a_1}{1+\cfrac{a_2}{1+\cfrac{a_3}{1+\ddots}}}$$

are $S_n(0) = s_1 \circ s_2 \circ \cdots \circ s_n(0)$.

It has become fashionable to consider so-called *modified* continued fractions where the truncation 0 is replaced by a modifying factor z_n, i.e., to study $S_n(z_n)$ ([**24,5,6,10**] and many other papers). This has been applied mostly for convergence acceleration but has been studied

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