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APPLICATIONS OF SZEGÖ POLYNOMIALS TO DIGITAL SIGNAL PROCESSING

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Dedicated to W.J. Thron on the occasion of his 70th birthday

ABSTRACT. Applications of Szegö polynomials, moment theory and two-point Padé approximants to problems in digital signal processing are described. The frequency analysis problem consists of determining unknown frequencies in a signal which is the sum of a finite number of cosine waves superimposed to white noise. The problem of filter design is to construct a causal filter T with finite energy, which has a prescribed amplitude response function $\Phi(\theta)$. Examples are given to illustrate each of the two applications.

1. Introduction. Connections between Szegö polynomials (orthogonal on the unit circle), the trigonometric moment problem and two-point Padé approximants are well known and have been given, for example, in [**2, 12, 13, 18, 19** and **21**]. The purpose of this expository article is to describe important applications of these topics to two problems involved with digital filters and the processing of digital signals.

In the frequency analysis problem, we consider a signal $u = \{u(k)\}$, superimposed on white noise, where $u(k)$ has the form

$$(1.1)\qquad u(k) = \lambda_0 + \sum_{j=1}^{I} \lambda_j \cos(\omega_j k + \varphi_j), \quad k = 0, \pm 1, \pm 2, \dots ,$$
$$1 \leq I < \infty, \quad \lambda_0 \geq 0, \quad \lambda_j > 0, \quad \omega_j, \varphi_j \in \mathbf{R} \quad \text{for } 1 \leq j \leq I.$$

We wish to determine the unknown frequencies $\omega_1, \omega_2, \dots , \omega_I$. The linear prediction method of Wiener [**28**] and Levinson [**24**] used for this problem is described in Section 3. Also included there for illustration

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