

## BIORTHOGONAL LAURENT POLYNOMIALS WITH BIORTHOGONAL DERIVATIVES

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**ABSTRACT.** If  $V_n$  and  $Y_n$  are monic polynomials of degree  $n$  with  $V_n(0) \neq 0$  and  $Y_n(0) \neq 0$  such that  $\{V_n(z), z^{-n-1} \cdot Y_n(z)\}_{n=0}^{\infty}$  is a biorthogonal system (BOS) with respect to a moment functional  $\Phi$  on the algebra of Laurent polynomials in  $z$  with complex coefficients, then  $\{V_n(z), z^{-n-1}Y_n(z)\}_{n=0}^{\infty}$  is called a *regular* BOS.

It is shown that if  $\{V_n(z), z^{-n-1}Y_n(z)\}_{n=0}^{\infty}$  and  $\{(1/(n+1))V'_{n+1}(z), z^{-n-1}(1/(n+1))Y'_{n+1}(z)\}_{n=0}^{\infty}$  are regular BOSs, then  $\{V_n(z), z^{-n-1}Y_n(z)\}_{n=0}^{\infty}$  is a so-called *classical* BOS, i.e., one of the systems of Examples 1–3 below. In this way we obtain a characterization of classical BOSs of Laurent polynomials, analogously to Hahn's [3] characterization of classical ordinary polynomials.

**1. Introduction.** It is well known that the derivatives of the classical polynomials, including the Bessel polynomials, are again orthogonal polynomials. In 1935, W. Hahn [3] showed that this property is characteristic for the classical orthogonal polynomials with positive weight function on a real interval. A few years later, in 1938, H.L. Krall [6] observed that Hahn's procedure also applies in the case of generalized orthogonality, i.e., orthogonality with respect to an eventually indefinite moment functional. Krall showed that the only generalized orthogonal polynomial systems with generalized orthogonal derivatives are, apart from a linear change of variable, the classical generalized orthogonal polynomials.

In the present paper we give a similar characterization of a certain class of biorthogonal systems of Laurent polynomials which is closely related to the class of the classical orthogonal Laurent polynomials as treated in [4], where a characterization of classical orthogonal Laurent polynomials in terms of second order differential equations is given (see [2] for ordinary polynomials).

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