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UNIFORM CONVERGENCE OF POLYNOMIALS ASSOCIATED WITH VARYING JACOBI WEIGHTS

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Dedicated to Prof. W.J. Thron on the occasion of his 70th birthday

ABSTRACT. In this paper we determine the functions on $[-1,1]$ that are uniform limits of weighted polynomials of the form $(1-x)^{\alpha_n}(1+x)^{\beta_n}p_n(x)$, where $\deg p_n \le n$, $\lim_{n\to\infty} \alpha_n/n = \theta_1 \ge 0$ and $\lim_{n\to\infty} \beta_n/n = \theta_2 \ge 0$. Estimates for the rate of convergence are also obtained. Our results confirm a conjecture of Saff for $w(x) = (1-x)^{\theta_1}(1+x)^{\theta_2}$, when $\theta_1 > 0$, $\theta_2 > 0$, and extend previous results of G.G. Lorentz and M. v. Golitschek, and Saff and Varga for incomplete polynomials.

1. Introduction. The introduction of "incomplete polynomials" by G.G. Lorentz [**4**] in 1976 has led to an extensive study of polynomials with varying weights. Among the more recent results is the solution of Freud's conjecture [**5**], and strong asymptotics for a family of extremal polynomials associated with exponential weights on $\mathbf{R}$ [**7**]. The essential question which serves as the starting point for these investigations is the following:

Suppose $w : \mathbf{R} \to \mathbf{R}$ is a nonnegative weight function continuous on its support Σ. An important problem is the characterization of limit functions of sequences of weighted polynomials of the form

$$[w(x)]^n p_n(x), \quad n = 1, 2, \dots ,$$

where $p_n \in \mathcal{P}_n$, the collection of all algebraic polynomials of degree at most n.

Mhaskar and Saff [**8**] proved that the sup norm of $[w(x)]^n p_n(x)$ over Σ actually "lives" on some (smallest) compact set $\mathcal{S} \subset \{x \in \Sigma : w(x) \neq 0\}$ which is independent of n and p_n. The connection between this fundamental result and our problem is that, in several important cases,

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