ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 21, Number 1, Winter 1991

THE RATIONAL INTERPOLATION PROBLEM REVISITED

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Dedicated to Wolfgang Thron on his 70th birthday

ABSTRACT. We present a general, homogeneous treatment of the rational interpolation problem in the extended complex domain. Interpolation conditions at the point ∞ and prescribed poles are allowed.

1. Introduction. Rational functions are functions meromorphic in the extended plane $\overline{\mathbf{C}}$. In $\overline{\mathbf{C}}$ the number ω of poles of a rational function $r \neq 0$ is equal to the number of its zeros; ω is called the *order* of r. We denote the set of rational functions of order at most ω by $\mathcal{R}_\omega$ and define the set $\overline{\mathcal{R}}_\omega := \mathcal{R}_\omega \cup \{\infty\}$ by adding the constant function ∞ to the set. (See Section 2 for more details.) Then, if t_1, t_2 are two Möbius transforms (i.e., meromorphic one-to-one maps of $\overline{\mathbf{C}}$ onto itself), the composition $t_1 \circ r \circ t_2$ is a rational function of the same order. Hence, in the theory of rational functions there is nothing special about the point ∞, neither in the domain nor in the range. However, in the usual treatment of the rational interpolation problem (or multipoint Padé or Newton-Padé approximation problem—as it is also called) one assumes that the prescribed function values are finite, and one treats the cases where data are given at the point ∞, or poles are prescribed, completely independently. As a consequence, the resulting theory lacks symmetry. For example, the result that r is the interpolant of f if and only if $1/r$ is the interpolant of $1/f$ does not always hold, since $1/f$ may take the value ∞. Also, while the rational interpolant is often constructed as a terminating Thiele fraction, some Thiele fractions correspond to

1980 *Mathematics Subject Classification.* 41A21, 41A05, 65D05
Keywords and phrases. Rational interpolation, Newton-Padé approximation, multipoint Padé approximation.
Received by the editors on August 1, 1988, and in revised form on October 28, 1988.