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DIVERGENCE OF VECTOR-VALUED RATIONAL INTERPOLANTS TO MEROMORPHIC FUNCTIONS

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Dedicated to Professor W.J. Thron on his seventieth birthday

ABSTRACT. A theorem on convergence of row sequences of Padé approximants for functions which are meromorphic in a disk was established by de Montessus in 1902. Stahl, in 1976, proved a companion theorem on divergence of these sequences outside the disk. In 1984, we extended de Montessus's theorem to the case of convergence in a disk of a row sequence of vector-valued approximants (simultaneous Padé approximants). Here, we establish the associated divergence results for these approximants outside the disk under virtually identical conditions.

1. Introduction. The famous convergence theorem of R. de Montessus de Ballore [**12**] for a row sequence of Padé approximants has become a result of considerable interest and generalization. Recently, attention has been directed to theorems containing generalizations of the interpolation set [**13, 20, 21, 18**], to inverse problems [**4, 5, 9, 2**], to associated divergence results [**14, 17, 18**] and to the adaptability of the theorem to vector-valued rational interpolation [**11, 6, 16**].

We discuss here the scheme of vector-valued rational interpolants whose confluent forms are also known as *simultaneous Padé approximants*. The polynomials constituting the vector-valued rational interpolants are also familiar as solutions of the *German polynomial approximation problem* [**10**]. Simultaneous Padé approximation involves approximation of several functions $\{f_i(z)\}_{i=1}^d$ which are analytic at $z = 0$ by rationals of the form $\{P_{N,i}(z)/Q_N(z)\}_{i=1}^d$, where the denominator polynomial $Q_N(z)$ is common to each of the d components. Let nonnegative integers $\rho_1, \rho_2, \ldots, \rho_d$ be given, such that $\sum_{i=1}^d \rho_i = M$. It is well known that nontrivial polynomials $\{P_{N,i}(z)\}_{i=1}^d$ and $Q_N(z)$

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