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THE USE OF REPULSIVE FIXED POINTS TO ANALYTICALLY CONTINUE CERTAIN FUNCTIONS

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Dedicated to Professor W.J. Thron on the occasion of his 70th birthday

Introduction. Attractive and repulsive fixed points can be used to enhance the convergence behavior of certain sequences of analytic functions that display the following compositional structure:

Let $\{f_n(\zeta, z)\}$ be a sequence of functions that are analytic in both variables (and continuous, along with their partials) in $S \times D$, where S and D are regions (not necessarily bounded) and, for each n, $D \supset f_n(S, D)$. Suppose that $f_n \to f$ on $S \times D$. Set

(1) $$F_1(\zeta, z) = f_1(\zeta, z) \text{ and } F_n(\zeta, z) = F_{n-1}(\zeta, f_n(\zeta, z)) \text{ for } n > 1.$$

The sequence

(2) $$\{F_n(\zeta, z)\}$$

may be called "limit periodic," an expression widely used to designate an important class of continued fractions that can be interpreted in this fashion.

The first investigation of the convergence behavior of such sequences (with regard to a fixed ζ) appears to have been a paper by Magnus and Mandell [**10**] on limit periodic compositions of linear fractional transformations (LFTs). They deduced that, in the most common circumstances, the sequence (2), in effect, converges to a common function $\lambda(\zeta)$ for all values of z except the repulsive fixed point (β) of the limit function. In particular, (2) converges for $z = \alpha$, the attractive fixed point of f. The author carried on these investigations by focusing on sequences $\{f_n\}$ of more esoteric varieties of LFTs [**1**]. Later, the author described the use of $z = \alpha$ to accelerate the convergence of certain limit periodic continued fractions (that may

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