ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 21, Number 1, Winter 1991

# MORE ON C-FRACTION SOLUTIONS TO RICCATI EQUATIONS

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ABSTRACT. An algorithm due to Merkes and Scott for finding $C$-fraction solutions of certain Riccati differential equations is generalized to apply to a larger class of Riccati equations. Remarks on computational aspects of the algorithm are made and several examples are presented.

**1. Introduction.** Continued fractions have been used theoretically to solve Riccati differential equations for many years. Euler [**6, 7**] and Lagrange [**11**] seem to be the originators of this approach. They both concentrated on very special forms of Riccati equations and found $C$-fraction solutions. More recently, Merkes and Scott [**12**], Stokes [**13**], Fair [**8**], Chisolm [**3**], Cooper [**5**], and Cooper, Jones and Magnus [ **4**] have made contributions using various continued fractions.

Riccati equations have the special property that they are invariant (in a sense) under linear fractional transformations (lfts). More precisely, under an lft,

$$y = \frac{\alpha(z)w + \beta(z)}{\gamma(z)w + \delta(z)}, \tag{1.1}$$

a Riccati equation,

$$y' = f_0(z) + f_1(z)y + f_2(z)y^2, \tag{1.2}$$

is transformed into another Riccati equation

$$w' = \tilde{f}_0(z) + \tilde{f}_1(z)w + \tilde{f}_2(z)w^2. \tag{1.3}$$

* This work supported in part by the National Science Foundation under Grant # DMS-8700498
Received by the editors on November 11, 1988, and in revised form on February 24, 1989.