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THE ONE-QUARTER CLASS OF ORTHOGONAL POLYNOMIALS

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ABSTRACT. An important class of orthogonal polynomials consists of those which satisfy a three-term recurrence relation with unbounded coefficients which have certain convergence properties. This paper reviews some of the known spectral properties of this class of polynomials. In order to put the discussion in the proper perspective, it includes an expository survey of the current knowledge of spectral properties of orthogonal polynomials in general as predictable on the basis of the behavior of the coefficients in the three-term recurrence relation.

1. Introduction. An important class of orthogonal polynomials consists of those whose three-term recurrence relation,

$$\begin{aligned} P_n(x) &= (x - c_n)P_{n-1}(x) - \lambda_n P_{n-2}(x), \\ P_{-1}(x) = 0, \quad P_0(x) &= 1, \quad c_n \text{ real}, \ \lambda_n > 0, \end{aligned} \tag{1.1}$$

have coefficients which satisfy the conditions

$$\lim_{n\to\infty} c_n = \infty, \quad \lim_{n\to\infty} \frac{\lambda_{n+1}}{c_n c_{n+1}} = \frac{1}{4} \tag{1.2}$$

Here we have assumed without loss of generality that our polynomials are monic. The classical prototype of this class is, of course, the sequence of Laguerre polynomials. There are a large number of natural questions concerning the spectral properties of the orthogonal polynomials of this class. For, under the hypotheses (1.2), it is possible for the zeros of the corresponding orthogonal polynomials to (i) form a dense subset of the interval $(0, \infty)$, (Laguerre polynomials, Wilson's continuous dual Hahn polynomials), (ii) have a derived set that forms a sequence converging to ∞ (certain Al-Salam and Carlitz polynomials, Askey-Ismail polynomials), (iii) spread from $-\infty$ to $+\infty$. The existence

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