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LIMITING BEHAVIOR OF SOLUTIONS OF $\mathbf{u_t = \Delta u^m}$ as $\mathbf{m \to \infty}$

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Consider the Cauchy problem for the porous medium equation

$$u_t = \Delta(|u|^{m-1}u) \qquad x \in \mathbf{R}^N, \quad t > 0 \tag{0.1}$$
$$u(x,0) = f(x) \qquad x \in \mathbf{R}^N. \tag{0.2}$$

We are interested in the behavior of the solution u as $m \to \infty$, for a fixed initial function f. Some study of this question was first carried out by Elliott, Herrero, King and Ockendon [**4**].

Under various conditions on f we will see that for fixed $t > 0$

$$u_m(\cdot, t) \to u_\infty \quad \text{as } m \to \infty \tag{0.3}$$

where u_m denotes the solution of (0.1)–(0.2), and $u_\infty = u_\infty(x)$ satisfies the "differential inclusion"

$$u_\infty - \Delta\varphi_\infty(u_\infty) \ni f. \tag{0.4}$$

Here φ_∞ is the maximal monotone graph

$$\varphi_\infty(s) = \begin{cases} 0, & |s| < 1 \\ \pm[0,\infty), & s = \pm 1 \\ \varnothing, & |s| > 1 \end{cases} \tag{0.5}$$

and the meaning of (0.4) is that there exists a function $w = w(x)$ such that

$$w(x) \in \varphi_\infty(u_\infty(x)) \quad \text{a.e.} \quad \text{and} \quad u_\infty - \Delta w = f \quad \text{in } \mathcal{D}'(\mathbf{R}^N). \tag{0.6}$$

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