# PHASE FIELD MODELS AND
# SHARP INTERFACE LIMITS:
# SOME DIFFERENCES IN SUBTLE SITUATIONS

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**1. Introduction.** We discuss a system of equations which has been investigated extensively [1–25]. In this article we compare this approach with various macroscopic formulations obtained as modifications of Stefan problems and indicate critical situations in which the two approaches differ. The crux of this approach involves an "order" parameter, $\phi$, which is coupled to the temperature, $u$, but is not simply dependent on it. Such a function $\phi$ can be expected to be a minimizer of the free energy such as

$$(1.1) \qquad \mathcal{F}\{\phi\} = \int_\Omega dx \{\xi^2 (\nabla \phi)^2 + \frac{1}{8a}(\phi - \phi^3) - 2u\phi\}$$

if the material occupies a region $\Omega$ and is in equilibrium. This is a consequence of a statistical mechanical analysis of a phase transition [25]. If the system is not in equilibrium, or in a steady-state, the free energy will no longer be a minimum, but will satisfy $\tau \phi_\tau = -\delta \mathcal{F}/\delta\phi$. The microscopic parameters $\tau, \xi$ can be related to macroscopically measurable quantities.

Coupled with a heat conservation equation which incorporates the latent heat of fusion, one then has the system, for any symmetric double-well potential $G(\phi)$ with minima at $\pm 1$,

$$(1.2) \qquad u_t + \frac{l}{2}\phi_t = K\Delta u$$

$$(1.3) \qquad \tau\phi_t = \xi^2 \Delta\phi + \frac{1}{2a}G'(\phi) + 2u$$

subject to appropriate initial and boundary conditions, e.g.,

$$(1.4) \qquad u(0,x) = u_0(x), \qquad \phi(0,x) = \phi_0(x), \qquad x \in \Omega$$

$$(1.5) \qquad u(t,x) = u_\partial(x), \qquad \phi(t,x) = \phi_\partial(x), \qquad x \in \Omega.$$