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A METHOD FOR THE NUMERICAL SOLUTION OF A CLASS OF NONLINEAR DIFFUSION EQUATIONS

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ABSTRACT. We propose a finite difference scheme for the diffusion equation, $(*)$ $u_t = d(u)\Delta u + f(u)$, on a general spatial domain of $\mathbf{R}^m$, $m \geq 1$, $d(u)$ is a bounded positive smooth function.

For the numerical solution of $(*)$ one usually uses a finite difference method based on the well-known θ-method, which requires a factorization of a matrix at each time step.

Here we propose a numerical scheme in which we need a single factorization of a matrix for each time level.

We prove that if W is an invariant region for $(*)$, it is also invariant for the proposed method. Comparisons between our scheme and the explicit/implicit Euler method are made.

We give an error bound which implies the first order convergence of the method and shows that the error does not exceed diam(W) for $t \to +\infty$. Finally, we show a numerical application.

1. Introduction. In recent years much interest has been shown in the numerical solution of the nonlinear diffusion equation

$$u_t = d(u)\Delta u + f(u), \qquad \Omega \times (0, +\infty) \tag{1.1}$$

with the addition of certain initial and boundary conditions. In (1.1), Ω is a bounded domain of $\mathbf{R}^m$, $m \geq 1$ with smooth boundary $\partial\Omega$; Δ denotes the Laplace operator in the spatial variable x and $d(\cdot)$ is a positive smooth function. Let $W = [0, b]$ be the u-space interval and let us suppose that $d(u)$ is bounded on W, that is, two positive constants α and β exist such that

$$0 < \alpha < d(u) < \beta \quad \text{for any } u \in W. \tag{1.2}$$

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