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PSEUDO-CONVERGENCE IN NORMED LINEAR SPACES

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A bounded sequence $\{x_n\}$ in a Banach space X is said to pseudo-converge to a point x_0, called the pseudo-limit, if x_0 minimizes the function $f_S(x) = \limsup ||y_m - x||$, $x \in X$, for every subsequence $S = \{y_m\}$ of $\{x_n\}$. If the pseudo-limit is unique, we call the sequence $\{x_n\}$ uniquely pseudo-convergent. This notion of convergence arose in connection with fixed point theory of multivalued nonexpansive mappings, see, e.g., [**1, 2, 4, 5, 7, 10**]. The basic idea is that if x_n is a bounded sequence of approximate fixed points of a multivalued nonexpansive mapping T, then there may exist a uniquely pseudo-convergent subsequence of x_n whose pseudo-limit is a fixed point of T.

In this paper we characterize pseudo-convergence in certain Banach spaces. A main result is that, in a space with a uniformly Gateaux differentiable norm, a sequence $\{x_n\}$ pseudo-converges to x if and only if $J(x_n - x)$ converges weak*ly to 0, where J is a duality map. We also consider other related types of convergence. Note that spaces with a uniformly Gateaux differentiable norm have appeared in several other contexts, including semigroups and approximations, see, e.g., Klee [**3**], Reich [**9, 11, 12**], Zizler [**13, 14**].

A space is said to satisfy (w^*)-Opial's condition if the condition $\{x_n\}$ converging weakly (weak*ly) to x_0 implies that

$$\limsup ||x_n - x_0|| < \limsup ||x_n - y||$$

for all $y \neq x_0$. Examples of spaces satisfying Opial's condition include Hilbert spaces and ℓ_p, $1 \leq p < \infty$. $L^p[0,1]$ for $p \neq 2$ do not satisfy the condition [**8**]. ℓ_1 satisfies w^*-Opial's condition.

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