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ℵ-PROJECTIVE SPACES IN NONCOMPACT CATEGORIES

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ABSTRACT. Neville and Lloyd have defined ℵ-projective topological spaces and characterized them in the category of compact Hausdorff spaces and continuous maps. The present paper characterizes the spaces ℵ-projective in various noncompact categories of topological spaces and maps.

1. Introduction. A topological space X is *projective* in a category provided whenever $g : X \to Z$ and $f : Y \to Z$ are admissible maps with f onto, a map $\psi : X \to Y$ can be found with $f \circ \psi = g$. Thus, the requirement is precisely that a solution ψ can be found making diagram (1) below commutative.

$$
\begin{tikzcd}
X \arrow[r, dashed, "\psi"] \arrow[dr, "g"'] & Y \arrow[d, "f\ (\text{onto})"] \\
& Z
\end{tikzcd}
\tag{1}
$$

Let $\aleph$ be an infinite cardinal. In [**16**] Neville and Lloyd defined a space to be $\aleph$-*projective* (in the category of compact Hausdorff spaces and continuous maps) provided diagram (1) has a solution ψ whenever all spaces are compact Hausdorff and the weight of Y is less than $\aleph$. They then showed that a compact Hausdorff space X is $\aleph$-projective if and only if it is a totally disconnected $F_\aleph$-space. (A space is an $F_\aleph$-space if and only if disjoint $\aleph$-open sets have disjoint closures; a set is $\aleph$-open if it is the union of fewer than $\aleph$ cozero sets.)

Our purpose here is to study $\aleph$-projectivity in various categories in which the objects are not necessarily compact. For this purpose, we will modify the definition of an $\aleph$-projective space by requiring that the weight of Z in diagram (1) also be less than $\aleph$. The resulting

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