ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 22, Number 2, Spring 1992

SOME NONCOMPACT HYPERSPACES WITH THE ALMOST FIXED POINT PROPERTY

SAM B. NADLER, JR.

ABSTRACT. It is shown that for certain types of subsets Z of arc-like continua, the space of all nonempty subcontinua of Z has the almost fixed point property. Our results generalize the well-known theorem that the space of all nonempty subcontinua of an arc-like continuum has the fixed point property. An example is given to show that our results do not extend to circle-like continua. Proofs are based on the use of the relatively new notion of almost universal mappings.

1. Introduction. A metric space (Y, d) is said to have the *almost fixed point property* (AFPP) provided that if $f : Y \to Y$ is a mapping (= continuous function), then, for each $\varepsilon > 0$, there exists $y_\varepsilon \in Y$ such that $d(f(y_\varepsilon), y_\varepsilon) < \varepsilon$. We note that for compact metric spaces, AFPP and the fixed point property are equivalent. Also, AFPP is preserved by uniformly continuous homeomorphisms (but not by homeomorphisms in general).

In [**1, 2, 15**] certain metric spaces are shown to have AFPP. We remark that the techniques in this paper are different than those in [**2, 15**] in that we use the notion of almost universal mappings which was defined and studied in [**1**].

A *continuum* is a nonempty compact connected metric space. For a continuum X, $C(X)$ denotes the space of all (nonempty) subcontinua of X with the Hausdorff metric [**9**]; if $Z \subset X$, then $C(Z) = \{A \in C(X) : A \subset Z\}$. An *arc-like* (*chainable*) *continuum* is a continuum X such that for each $\varepsilon > 0$, there is an ε-mapping from X onto $[0, 1]$. In [**14**] it was shown that if X is an arc-like continuum, then $C(X)$ has the fixed point property. In this paper we generalize this result by showing that $C(Z)$ has AFPP for certain types of subsets Z of arc-like continua. If X is a circle-like continuum, then $C(X)$ has the fixed point

Received by the editors on August 5, 1988, and in revised form on July 7, 1989.
AMS subject classification. Primary 54B20, 54H25, Secondary 54F20.
Key words and phrases. Almost fixed point property, almost universal mapping, arc-like continuum, circle-like continuum, Hausdorff metric, solenoid, universal mapping.