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A UNIFORM GEOMETRIC PROPERTY OF BANACH SPACES

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ABSTRACT. Property (β) of Banach spaces was introduced by S. Roiewicz as an intermediate property between uniform convexity and nearly uniform convexity. It is proved in this note that there are Banach spaces with property (β) which cannot be renormed in a uniform convex manner. This answers a question in [**8**].

1. Notations. We follow standard terminology which can be found in [**3**] or [**6**]. Let $(X, ||\cdot||)$ be a Banach space. B_X denotes its closed unit ball, $\operatorname{conv}(A)$ ($\overline{\operatorname{conv}}(A)$) is the convex hull (the closed convex hull) of a subset A of X. If (x_n) is a sequence in X, let $\operatorname{sep}(x_n) = \inf\{||x_n - x_m|| : n \neq m\}$. $\mathbf{K}$ denotes the field of the real or complex numbers.

2. Introduction. Several classes of Banach spaces have been introduced in the past according to the fulfillment of certain uniform properties. We can mention:

(UC): Uniform convexity (Clarkson [**1**]): $\forall \varepsilon > 0\, \exists \delta > 0$ such that if $x, y \varepsilon B_X$ and $||x - y|| > \varepsilon$, then

$$||\frac{x+y}{2}|| < 1 - \delta.$$

(NUC): *Nearly Uniform convexity* (Huff [**5**]): $\forall \varepsilon > 0\, \exists \delta > 0$ such that, if (x_n) is a sequence in B_X with $\operatorname{sep}(x_n) > \varepsilon$, then $\operatorname{conv}(x_n) \cap (1-\delta) B_X \neq \phi$.

(β): Property (β) (Rolewicz [**8**]): Given an element $x_0 \in X \sim B_X$ define the associated *drop* $D(x_0, B_X)$ as the set $\operatorname{conv}(B_X \cup \{x_0\})$,

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