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PULLBACKS OF BANACH BUNDLES

J.W. KITCHEN AND D.A. ROBBINS

ABSTRACT. Let S and T be compact Hausdorff spaces, $\alpha : S \to T$ a continuous map, and $\rho : F \to T$ a Banach bundle. The bundle $\pi : E \to S$ which is the pullback to S of ρ via α, has fibers given by $E_p = F_{\alpha(p)}$ for $p \in S$. The present paper investigates some properties, such as norm continuity and Hausdorfness, which the bundle π inherits from ρ via the map α. The main result presents a relation between the section spaces $\Gamma(\rho)$ and $\Gamma(\pi)$ via inductive tensor products.

This paper continues the study of Banach bundles which has been pursued by the authors in previous papers [**2, 3, 4, 5, 6**], to which the reader is referred for notations, definitions, and general information. This time the focus is upon pullback bundles. If $\pi : F \to T$ is a bundle of Banach spaces, and if $\alpha : S \to T$ is a continuous map, then there is a bundle of Banach spaces $\pi : E \to S$, called the *pullback* of the given bundle by α, with the following properties

1) if $p \in S$, then $E_p = \pi^{-1}(\{p\})$, the stalk in the pullback bundle over p, is an isomorphic copy of $F_{\alpha(p)} = \rho^{-1}(\{\alpha(p)\})$;

2) If σ is any section in $\Gamma(\rho)$, then its pullback by α, namely $\alpha^*(\sigma) = \sigma \circ \alpha : S \to E$ is a section in $\Gamma(\pi)$. (See Kitchen and Robbins [**3**] for a discussion of pullbacks.)

Our results are basically of three sorts. First, we investigate properties which the pullback bundle inherits from the given bundle. (If, for instance, the bundle $\rho : F \to T$ is norm continuous, is the pullback bundle $\pi : E \to S$ also norm continuous, is the pullback bundle $\pi : E \to S$ also norm continuous?) The main result (Theorem 5) presents a relation between the section spaces $\Gamma(\rho)$ and $\Gamma(\pi)$ via inductive tensor products. The substance of the theorem is summarized by the equation

$$C(S)\hat{\hat{\otimes}}_{C(T)}\Gamma(\rho) \cong \Gamma(\pi),$$

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