ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 22, Number 2, Spring 1992

POLYNOMIAL INTERPOLATION OF HOLOMORPHIC FUNCTIONS IN C AND $\mathbf{C}^n$

T. BLOOM *, L. BOS, C. CHRISTENSEN AND N. LEVENBERG

0. Introduction. This paper is concerned with polynomial interpolation, particularly Lagrange interpolation, of functions holomorphic in a neighborhood of a polynomially convex nonpluripolar compact set $K \subset \mathbf{C}^n$. The general framework is as follows: let $h_d = h_d(n) \equiv \binom{n+d}{d}$ denote the dimension of the complex vector space P_d of holomorphic polynomials of degree at most d in n variables $z_1, \dots, z_n$. For each positive integer d, we choose h_d points $A_{d1}, \dots, A_{dh_d}$ in K; thus we get a doubly-indexed array $\{A_{dj}\}_{\substack{j=1,\dots,h_d \\ d=1,2,\dots}}$ of points in K. Given a function f holomorphic in a neighborhood of K, we would like to know under what conditions on the array do the Lagrange interpolation polynomials $L_d f$ associated to f and $\{A_{dj}\}$ converge uniformly to f on K. In one variable ($n = 1$), Walsh [**23**] proved a necessary and sufficient condition on the array $\{A_{dj}\}$ in order to guarantee uniform convergence of the sequence $\{L_d f\}$ to f on K for each such f (see 1.4). In theorem 1.5 we give several other conditions—they are not equivalent to Walsh's condition and in theorem 1.5 we give the precise relation between the various conditions.

The proof of Walsh's condition depends on the Hermite remainder formula (1.3). No such simple formula is available in the case $n > 1$. We will show, via several examples, that many analogues of the one variable results do not hold. Theorem 4.1 summarizes our knowledge of the generalization of Theorem 1.5 to several variables. The results are not as complete as the one variable case (see, in particular, Problem 5.5).

Finally, in Section 5 we list a few open questions on polynomial interpolation in $\mathbf{C}^n$.

1. One-variable case. 1.1 Let $K \subset \mathbf{C}$ be compact, nonpolar, and polynomially convex. For simplicity, we assume K is regular for the

Received by the editors on February 1, 1989.
* Partially supported by NSERC grant A7535.