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PRIME IDEALS AND FINITENESS CONDITIONS FOR GABRIEL TOPOLOGIES OVER COMMUTATIVE RINGS

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ABSTRACT. We examine some finiteness conditions on Gabriel topologies, particularly over commutative rings. We study special classes of topologies defined by sets of prime ideals and provide examples to establish that a number of these classes are distinct.

Introduction. The equivalent concepts of Gabriel topologies, hereditary torsion theories, idempotent radicals and idempotent kernel functors, were originally investigated, starting about 1960, with an eye to extending localization techniques, so useful in commutative algebra, to the realm of noncommutative rings. However, relatively early in the study of Gabriel topologies, in 1973, P.J. Cahen noted there were interesting questions about such topologies even in the context of commutative rings. More recently, other investigators have focused on this context, looking at either Gabriel topologies (e.g., [**2, 3, 5, 13**]) or idempotent kernel functors (e.g., [**4, 19, 20**]). This paper is related to work done by these people.

Finiteness conditions of one sort or another, either on Gabriel topologies, or on the underlying ring, play a crucial role in the work cited above. So do prime ideals. One indication of the connection between finiteness conditions and prime ideals is that if R is a *noetherian* commutative ring, then every Gabriel topology on R is determined by a set of prime ideals. It is not surprising that many of the finiteness conditions on Gabriel topologies referred to in the literature imply that the topology is determined by prime ideals. The investigation of topologies determined by prime ideals will be the primary goal of this paper, and the focus of Sections 1 and 2.

In studying Gabriel topologies over commutative but not noetherian rings, it is often necessary to restrict the topologies in order to obtain

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