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A TOPOLOGICAL APPROACH TO MORITA EQUIVALENCE FOR RINGS WITH LOCAL UNITS

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ABSTRACT. In [**1**] and [**3**] a theory of Morita equivalence has recently been developed for certain not necessarily unital rings called rings with local units. In this article we prove that the special Hom-sets which figure in the description of equivalence functors are actually the sets of continuous homomorphisms from a locally projective generator (endowed with a suitable topology) into discrete modules. The main result of this paper says that two rings with local units which fulfill a topological condition of projectivity are Morita equivalent if and only if suitable matrix rings over them are isomorphic to each other.

Following the terminology of [**3**], a ring R is said to have *local units* if there is a set E of idempotents in R such that for any $r, s \in R$ there is an $e \in E$ which acts as a two-sided identity for both r and s; in particular, any unital ring has local units with $E = \{1\}$. Note that if R has local units, then $R = \cup_{e \in E} eRe$. For any ring R with local units and any (infinite) set I, denote by R_I^f the ring of $I \times I$ matrices over R which contain at most finitely many nonzero entries. Clearly, R_I^f also has local units.

Throughout this article all modules are assumed to be left modules (unless otherwise indicated), and all module homomorphisms will be written on the right. For any set I and any module M, we denote by $M^{(I)}$ the (discrete) direct sum of I copies of M.

Let R be a ring with local units. As in [**3**], we denote by RMod the category of unitary modules ${}_RM$ (those with $RM = M$) together with all R-homomorphisms. Recall [**3**, Definition 2] that a module $P \in R$Mod is said to be *locally projective* if there is a direct system $\{P_i\}_{i \in I}$ of finitely generated projective direct summands of P (so that

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