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MEAGER-NOWHERE DENSE GAMES (I): n-TACTICS

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ABSTRACT. In the introduction to this article we give a brief survey of a problem in the theory of Banach-Mazur games. We introduce two games, $MG(J)$ and $SMG(J)$ (where J is a free ideal on some set), which evolved from a study of an example relevant to this problem. The second player has a winning perfect information strategy in both of these games and we examine under what conditions it suffices for the second player to remember only the most recent n or fewer moves of the opponent (n some fixed positive integer) in order to insure a win. Strategies depending on only this information are called n-tactics.

The subject of this article belongs to the areas of combinatorial games and of topological games of length ω. In this rather lengthy introduction we give a short survey of the problem that motivated the work to be presented here. Readers who are interested in more details could consult Telgarsky's survey paper [**11**] and its extensive bibliography to the source literature.

The Scottish Book [**14**, Prob. 3] is probably the earliest popular record of the Banach-Mazur game. This game on a topological space (X, τ) is denoted by $BM(X, \tau)$ and is played as follows. First, player ONE picks a nonempty open subset E_1 of X, after which TWO picks a nonempty open subset N_1 of E_1. Next, ONE picks a nonempty open subset E_2 of N_1 and TWO responds with a nonempty open subset N_2 of E_2, and so on. In this manner, the players construct a sequence $(E_1, N_1, \dots, E_k, N_k, \dots)$ where for each positive integer k,

(i) E_k denotes ONE's k'th move and N_k, TWO's k'th move.

(ii) E_{k+1} is a subset of N_k which in turn is a subset of E_k, and these are all nonempty open subsets of X.

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