ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 22, Number 3, Summer 1992

MELNIKOV'S METHOD, STOCHASTIC LAYERS AND NONINTEGRABILITY OF A PERTURBED DUFFING-OSCILLATOR

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ABSTRACT. Melnikov's method is used to show that the perturbed Duffing-oscillator with Hamiltonian

$$H_\varepsilon(q,p,x,y) = \frac{p^2-q^2}{2} + \frac{q^4}{4} + \frac{x^2+y^2}{2} + \varepsilon q(p-y)$$

has a subharmonic of order m provided $\varepsilon \neq 0$ is sufficiently small and $h_0 > h_m$ where $h_0 = H_0(q,p,x,y)$ is the total energy of the unperturbed Duffing-oscillator and h_m is the energy of the resonant periodic orbit of order m of Duffing's equation; furthermore, for $\varepsilon \neq 0$ sufficiently small and $h_0 > 0$, this system is nonintegrable and there is a primary stochastic layer of width

$$d = \frac{2\pi\varepsilon\sqrt{h_0}\,\mathrm{sech}\,(\pi/2)}{\sqrt{p_0^2 + q_0^2(1-q_0^2)^2}} + O(\varepsilon^2)$$

near a point $(q_0,p_0) \neq (0,0)$ on the homoclinic manifold of the unperturbed system as well as resonant stochastic layers whose bandwidths $d_m = O(\sqrt{\varepsilon/m})$.

1. Introduction. The occurrence of stochastic regions in two-degree-of-freedom Hamiltonian systems has been a topic of mathematical interest for many years, from both a theoretical and an applied point of view. In their 1964 paper on the nonintegrability of galactic motions [**6**], Henon and Heiles presented a numerical study of a Hamiltonian system with

$$H(q,p,x,y) = \frac{p^2+q^2}{2} - \frac{q^3}{3} + \frac{y^2+x^2}{2} + x^2 q.$$

Their work clearly showed the appearance and evolution of stochastic regions with increasing energy levels. It also raised the question as to whether such behavior could be described analytically.

Received by the editors on October 9, 1988 and in revised form on February 20, 1989.